A Case Study of Applying a Novel Asset Maintenance Optimization Methodology to Electricity Distribution Utilities using Simulation Strengthened Analytics

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Abstract

Power Utilities are among the most capital-intensive industries in the world, where even modest improvements in asset maintenance and capital investment scheduling can potentially lead to very large annual cost savings in the order of millions. Such an improved scheduling model requires reliable asset condition data, historical asset data, SCADA data, etc. However, lack of sufficient visibility into the asset condition and insufficient, disjointed or mismanaged historical data records are the major challenges for this improved asset management solution.

In this paper an integrated electrical and optimization model will be presented, which optimizes the asset maintenance and capital investment schedules. Preliminary results have been achieved and tested on realistic data for optimizing transformer maintenance in the Netherlands. The results have shown that this approach can provide visibility into asset conditions, conduct forward-looking what-if analysis for better grid planning, and reduce the operating costs including the cost associated with asset failure significantly. This can translate to multi-million dollar savings in operational costs under a fixed asset management budget.

1 Introduction

As one of the largest Distribution System Operators (DSO) in the Netherlands, Alliander serves several million customers with electricity. In order to distribute the electricity reliably from the 150kV at the primary substations – connection point between Transmission System Operators (TSO) and DSO - down to the 0.4kV level at the residential homes, a large infrastructure consisting of several components such as underground cables, transformers, and switchgear needs to be maintained. Failure of these components, for example due to aging, may lead to power outages and hence unacceptable high SAIDI (System Average Interruption Duration Index) numbers.

Due to factors such as tighter budget constraints, aging components [1] and the energy transition, the need to calculate component remaining lifetime has become ever more important. Knowing the remaining lifetime of critical components will help asset managers to prioritise maintenance activities (OPEX) and capital investments (CAPEX), given a certain budget, which will reduce the risk of power outages.

Adopting well established models, such as the Weibull distribution, for determining the remaining lifetime of critical assets is common practice with many utilities. However, often these models are static and lack dynamic inputs such as the electrical states of the system. This paper presents a holistic approach to address the challenges for an improved asset management solution by combining the strength of both power network simulation and big data analytics, which is an emerging technology within the utilities. The essential new elements to this approach are:

(1) A data-mining technique to build time and space coherent data models from historical data records with a wide-range of time scales.
(2) A load model to provide an input for the asset model.
(3) A physical model based asset health condition assessment to quantify the risk associated of each asset based on its type, location, historical usage, and environment conditions.
(4) An effective algorithm that determines the optimal asset maintenance schedules (i.e., minimize the expected cost caused by asset failure) under constraints such as budget and personnel capacity.

Both the power transformers and distribution transformers, in total tens of thousands for Alliander, are crucial components in the grid where a failure of these components may result in an unacceptable increase in SAIDI. The impact of a failure of power transformers is usually more than it is the case with distribution transformers considering the fact that power transformers are functionally one level above distribution transformers.

Due to its criticality in the grid and significant associated maintenance and capital investment costs, transformers have been selected as the asset to test the holistic approach on. In this paper the focus will be mainly on distribution transformers. Applying the holistic approach to power transformers will be future work of the authors.

Zaltbommel, which is a municipality located in the south of the Netherlands, has been used as the case study location to develop and test the holistic optimization approach. As opposed to the rest of Alliander's secondary substations, several secondary substations within in Zaltbommel have been equipped with digital real-time measurements to measure the power, currents, and voltages. These measurements are both on the medium voltage (MV) and low voltage (LV) side of the distribution transformer and provide, in combination with grid models, a way to estimate all the electrical states in this grid. The flowchart of the approach in the paper is shown in Fig. 1.

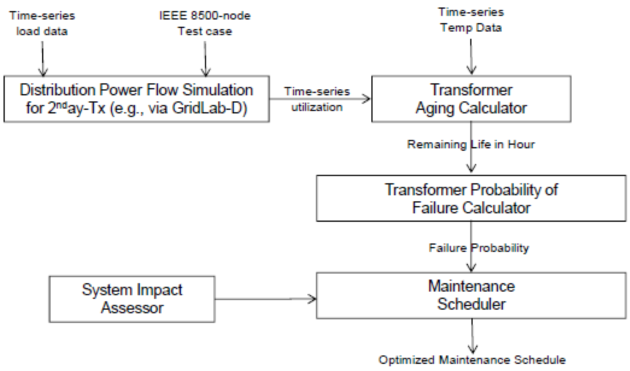

Fig. 1 Flowchart of the approach in the paper

2 Load Profile Model

One power transformer steps down the voltage from 150 kV to 10.5 kV. A number of MV feeders are connected to the MV busbar of the power transformer, as shown in Figure 2. Some feeders feed a regulating transformer that restores the voltage level back to 10.5 kV to make up for the voltage loss caused by the long distance of the MV cables while other feeders feed the power to the loads.

A MV feeder typically feeds of a number of distribution transformers, which steps down the voltage level from 10.5 kV to 0.4 kV. At the output bus of a distribution transformer, there are a number of LV feeders, which eventually supply power to residential homes or commercial buildings. Some customers are connected to one phase of the LV cables while some others are connected to the three phases, depending on the contracted power. Some large (e.g., industrial) customers may draw power from a MV feeder directly.

The obvious way of determining the load on a distribution transformer would be to obtain the LV electrical network of feeders and cables, with customers (residential, commercial or otherwise) being the end-point loads.

However, in the absence of detailed network connectivity data, some other means of attributing customer load to the cables and feeders and ultimately the distribution transformers needs to be devised. There are two approaches taken to tackle this issue based on geographical location data.

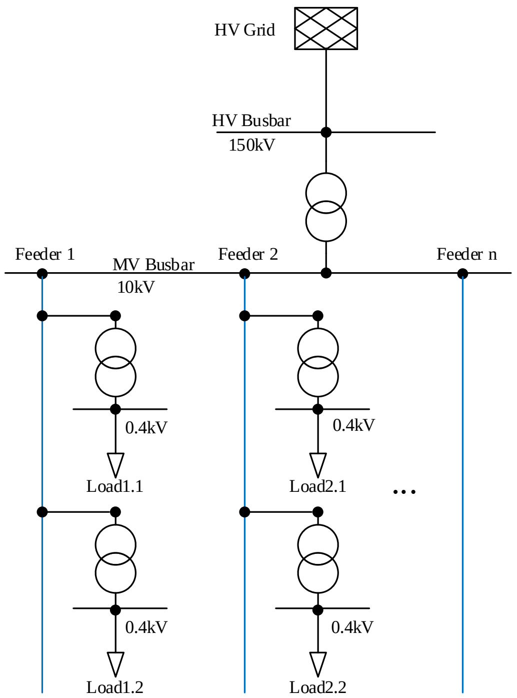

Fig 2 Simplified network structure of substation Zaltbommel1

1. *Cable length approach*

The first approach uses the availability of postal zone information for customers and cable sections. The common factor that can be used to associate customers and cables is the postcode. However, since it is possible to have several cables in a postcode area an assignment problem arises as to which customer feeds off which cable. The approach assumes that the number of customers connected to a given cable to be proportional to the length of that cable in that postcode area.

The customer to cable assignment (per postcode) can be done in multiple ways depending on the intended purpose:

- Reflect the true connectivity, i.e., the intention is to approximate the real-life situation as best as possible but doing an educated guess as to which customer connects to which cable. A reasonable guideline would be to make sure that the number of customers assigned to the cable is proportional to that cable segment's length in the postcode. This leaves open the choice of the identity of the customer and therefore the load attributed to the cable.
- Determine a balanced load. The objective is to arrive at a uniformly distributed load per cable. Note that load balancing is preserved when the load is aggregated over all postcode that the cable runs through. This invariant

simplifies the calculations.

- Deliberately skew the load. Here the intent is to arrive at a bad or worst-case situation This clearly causes an unbalanced load and unfortunately destroys the invariant mentioned above: if a certain cable (in a postcode) is selected to carry the bulk of the load, that same cable should play the same role in the other postcodes it runs through as well, hence we can no longer deal with postcodes independently.
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2. Connectivity approach

In this approach, the assignment of the customers is based on the postcode and the house number of the customers. Based on geographical coordinated of the cable, the cable route is mapped. The customers are then assigned to the nearest cable that passes the customer's address.

After the LV network of the cables is obtained, the loading of each cable, and eventually of the transformer, is the total load of the customers connected to the transformer over time. The load profile has a resolution of 15 minutes and incorporate not only the historical loading data but also a prognosis of the load in the future.

A power flow calculation is needed to calculate the voltage and current on each component, based on active and reactive power. Mathematically this is done via two sets of equations:

$$P = \sqrt{3}UIcos\phi \quad (1)$$

$$Q = \sqrt{3}UIsin\phi \quad (2)$$

Where

P	active power
Q	reactive power
U	line – line voltage
I	current
Φ	angle of powerfactor

But in MV grid the voltage deviation is very small and the power factor of loads is almost 1 [2]. Therefore in this paper only active power is considered for transformer aging. And since the time-series load curve are already known from the load profile model, power flow calculations are not needed for evaluation of transformer aging. This method simplifies the problem, but its accuracy is limited because it does not consider the voltage deviation.

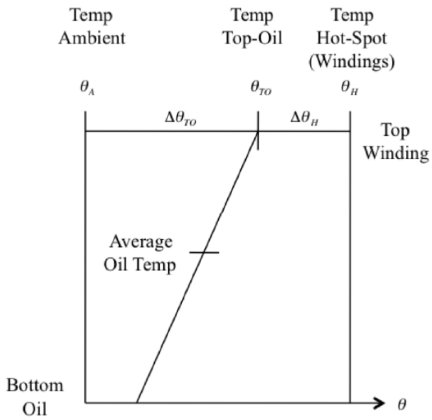
In the future, however, if the aging of other components such as cable or joint needs to be evaluated, the real-time current and voltage of those components should be known. Thus a time-series power flow calculation is necessary, which executes a number of steady-state power flow calculations at each time point of the load profile.

3 Transformer Aging

3.1 *Hotspot Temperature*

Transformers age at different rates based on the ambient temperature, load, cooling mechanism, and insulation type. Transformers which are lightly loaded and in geographic areas where ambient temperatures are low, will age at a slower rate than transformers which are heavily loaded and in regions of high ambient temperature.

Modeling transformer aging is done by solving the heat transfer equations for the top-oil temperature and the hot-spot temperature in the windings. The hot-spot temperature will degrade the paper insulation and causes irreversible damage. The thermal equations are first-order differentials equations described in [3], [4].



Hotspot temperature: θ_H

$$\theta_{\mathrm{H}} = \theta_A + \Delta\theta_{TO} + \Delta\theta_H \quad (3)$$

Top – oil temperature: $\Delta\theta_{TO}$

$$\Delta\theta_{TO,U} = \Delta\theta_{TO,R}\left(\frac{K^2R+1}{R+1}\right)^{\mathrm{n}}, \; K = \frac{I_{load}}{I_{rated}}$$

$$\Delta\theta_{TO,U} = \tau_{TO}\frac{d\Delta\theta_{TO}}{dt} + \Delta\theta_{TO} \quad (4)$$

$$\Delta\theta_{TO} = (\Delta\theta_{TO,U} - \Delta\theta_{TO,i})(1-e^{-\frac{t}{\tau_{TO}}}) + \Delta\theta_{TO,i}$$

Hot- spot temperature rise over top- oil: $\Delta\theta_H$

$$\Delta\theta_{H,U} = \Delta\theta_{H,R}K^{2m}$$

$$\Delta\theta_{H,U} = \tau_H\frac{d\Delta\theta_H}{dt} + \Delta\theta_H \quad (5)$$

$$\Delta\theta_H = (\Delta\theta_{H,U} - \Delta\theta_{H,i})(1-e^{-\frac{t}{\tau_H}}) + \Delta\theta_{H,i}$$

Where:

- θ_{A} ambient temperature

  $\Delta\theta_{\mathrm{TO}}$ top-oil temperature rise

  $\Delta\theta_{\mathrm{H}}$ winding hot-spot temperature rise

- $\Delta\theta_{TO,U}$ ultimate top-oil rise over ambient
  $\Delta\theta_{TO,R}$ top-oil rise over ambient at rated load
  K ratio of current load to rated current load
  R ratio of load loss to non-load loss
  n exponent for variation of Δ_{TO} with changes in load
  τ_{TO} time constant of top-oil
  $\Delta\theta_{TO,i}$ top-oil rise initial condition

- $\Delta\theta_{H,U}$ ultimate winding hot-spot rise over top-oil
  $\Delta\theta_{H,R}$ winding hot-spot rise over top-oil at rated load
  m exponent for variation of Δ_{H} with changes in load
  τ_{H} time constant of the windings
  $\Delta\theta_{H,i}$ winding hot-spot rise initial condition

3.2 *Accelerated Failure Time Model*

To predict the failure probability of a transformer, a Weibull distribution is trained to find the coefficients (η, β) based on failure times of the transformers. The failure probability is calculated by

$$F(t) = 1 - e^{-(\frac{t}{\eta})^{\beta}} \tag{6}$$

where t is the calendar age of the transformer. The Weibull parameter is the scale parameter and corresponds to the characteristic life. When the failure probability is

$$F(t=\eta) = 1 - e^{-1} = 0.632 \tag{7}$$

the electrical age is the time at which 63.2% of the transformers will fail.

With an accelerated failure time (AFT) model, time t is replaced by At the electrical age of the transformer.

$$F(At) = 1 - e^{-(\frac{At}{\eta})^{\beta}} \tag{8}$$

This determines the failure probability of a transformer based on the acceleration factor A. If $A > 1.0$, then the transformer has aged at an accelerated rate and may need to be replaced at an earlier date than the transformer's expected lifetime.

Computing failure probabilities accurately, need accurate computations of:

1. (η, β) using regression or maximum likelihood estimation from failure data
2. A, the acceleration factor, by solving heat transfer differential equations for the hot-spot temperature.

The Weibull hazard function which is useful for maintenance optimization is defined as:

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\beta}{\eta^{B}} t^{\beta-1}$$

4 Maintenance and Capital Investment Optimization

In this section, we develop an optimization framework to find the optimal maintenance (including replacement) schedules for a set of transformers under given budget and labour constraints. First, we introduce the problem formulation, then a solution approach to solve it is given. Since the size of the problem under consideration is very large, which consists of several binary variables in the model, it becomes intractable if we try to deal with the unified NP-hard nonlinear problem for the entire system. We attack this issue by separating into two stages: computing optimal maintenance intervals for each transformer independently without considering the global constraints, and then forming a scheduling problem based on the optimal values from the first step taking into account the budget and labour constraints.

4.1 *Optimized PM Intervals for Individual Unit*

The aim of preventive maintenance (PM) activities is to decrease the failure rate and/or the electrical age of a transformer. We assume that PM actions are performed at times $t_1, t_1 + t_2, \ldots, \sum_{k=1}^{K-1} t_k$ with a cost c_p for each PM and a possible replacement is at time $\sum_{k=1}^{K} t_k$ with a cost c_r during a fixed time horizon of interest $\bar{t}$. Denote $h_0(t)$ by the failure rate function, which is given from the previous section, if no PM intervention is conducted. The effect of PM at the k-th period is characterized by a hybrid model [5], [6], which models the failure rate function $h_k(t)$ between the (k-1)-th and the k-th PMs by changing from the previous time interval $h_k(t)$ as

$$h_k(t) = a_k h_{k-1}(b_k t_{k-1} + t), \tag{9}$$

where a_i, b_i are adjustment factors estimated by the PM historical data. Let y_k be the electrical age immediately before the k-th PM, thus we have

$$\begin{aligned} t_k &= y_k - b_{k-1} y_{k-1} \\ y_k &= t_k + b_{k-1} t_{k-1} + \ldots + b_{k-1} b_{k-2} \ldots b_1 x_1 \\ \sum_{i=1}^{k} t_i &= y_k + \sum_{i=1}^{k-1} (1 - b_{i-1}) y_{i-1}. \end{aligned} \tag{10}$$

The total failure (impact) cost is computed from $c_m \int_{b_{k-1}y_{k-1}}^{y_k} h_k(t)dt$. We make the following assumptions:

1. The failure rate function $h_0(t)$ is continuous and strictly increasing if there are no PM activities;
2. A minimal repair is carried out for a failure between PMs, and it only restores the function of the unit, but does not change the failure rate;
3. PM, repair, and replacement times are negligible;
4. A unit is "good-as-new" after a replacement.

We determine the optimal scheduled intervals t_k by minimizing the mean cost-rate, represented as a function with respect to the electrical age y_k, which is achieved by distinguishing two cases.

If there is no need to replace, the mean cost-rate optimization problem is formulated as

$$\min_{y,K} \; f_K^1(y) = c_p(K-1) + c_m \sum_{k=1}^{K} \int_{b_{k-1}y_{k-1}}^{y_k} h_k(t) \qquad (11)$$

s.t.

$$y_k - b_{k-1}y_{k-1} \geq 0, k = 1, \ldots, K \qquad (12)$$

$$y_K + \sum_{k=1}^{K-1} (1 - b_{k-1}) y_{k-1} = \bar{t} \qquad (13)$$

$$y_k \geq 0, k = 1, \ldots, K. \qquad (14)$$

The objective function in Equation (11) is the total cost of the unit over the fixed time horizon $\bar{t}$, including the cost for $K-1$ PMs and the expected cost of repairs for failures. We impose the positivity condition for the time intervals t_k in (12) and the electrical age y_k in Equation (14), respectively. Constraint in Equation (13) is the length of the scheduled horizon. Note that at time $\sum_{k=1}^{K} t_k = \bar{t}$, there may be no a PM. The mean cost-rate is equal to the objective function value in Equation (11) divided by $\bar{t}$. If the transformer needs to be replaced, the optimization problem becomes

$$\min_{y,K} \; f_K^2(y) = \frac{c_p(K-1) + c_r + c_m \sum_{k=1}^{K} \int_{b_{k-1}y_{k-1}}^{y_k} h_k(t)dt}{y_K + \sum_{k=1}^{K-1} (1 - b_{k-1}) y_{k-1}} \qquad (15)$$

s.t.

$$y_k - b_{k-1}y_{k-1} \geq 0, k = 1, \ldots, K \qquad (16)$$

$$y_K + \sum_{k=1}^{K-1} (1 - b_{k-1}) y_{k-1} \leq \bar{t} \qquad (17)$$

$$y_k \geq 0, k = 1, \ldots, K. \qquad (18)$$

The numerator of Equation (15) comprises the replacement cost and the denominator is the total life of the transformer. Constraint in Equation (17) requires a replacement before the time $\bar{t}$. We solve the two problems in Equations (11-14) and (15-18) simultaneously and select the PM schedule associated with that of a smaller mean cost-rate. We notice that above formulations have an integer variable K and continuous variables y_k's. The optimum number of PM intervals K is calculated by explicit enumeration. For a fixed K, they are nonlinear continuous optimization problems, which can be written as the following general format with a nonlinear objective function f for both problems:

$$\min_y \; f(y)$$
$$Ay \leq b,$$

where A is a matrix, b is a vector. Because $h_0(t)$ is continuous, hence f is differentiable. This optimization problem can be solved by a gradient projection algorithm [7], [8], where the projection is a linearly constrained least squares problem. Some efficient approaches which are specific to the problem have been developed [9]. The warm-start capability of the gradient projection algorithm is exploited when increasing the value of K.

4.2 *Optimized Maintenance and Replacement Schedule*

In practice, there are some operational and economic constraints for the entire transformer system such as labour and budget availability for each period. If the PM planning based on the solution from Section 4.1 is deployed, some of these constraints will be likely violated. Our scheduling strategy is to obey the global constraints for the final planning, but possibly readjust the optimal scheduled PM intervals for each transformer obtained from the first step. We assume that after a replacement, a brand-new unit with the same type is used. The continuous time horizon $\bar{t}$ is discretized into a finite evenly spaced time intervals I. In this model, we attempt to minimize the total deviation from the optimal PM intervals given from the first stage. The following notations are used in our formulation.

Parameters

- N set of transformers;
- I set of time intervals;
- K_n set of PMs and a possible replacement for the n-th transformer;
- r_n period for a replacement of the n-th transformer;
- w_n impact weight of the n-th transformer;
- $t^{k,n}$ optimal PM intervals for the n-th transformer obtained from the first stage;

γ_n manpower for a PM of the n-th transformer;

β_n manpower for a replacement of the n-th transformer;

θ_n cost for a PM of the n-th transformer;

ρ_n cost for a replacement of the n-th transformer;

h_i maximum manpower availability during the i-th interval;

m_i maximum budget availability during the i-th interval;

$i_l^{k,n}$ lower bound for the deviation of the k-th PM or a replacement of the n-th transformer;

$i_u^{k,n}$ upper bound for the deviation of the k-th PM or a replacement of the n-th transformer;

Variables

$x_i^{k,n}$ binary variable indicating whether the k-th PM or a replacement is conducted for the n-th transformer during the i-th interval.

We minimize an objective function of the total deviation

$$\min_{x_i^{k,n}\in\{0,1\}} \sum_{i\in I, n\in N, k_n\in K_n} w_n x_i^{k_n,n} \, |i - t^{k_n,n}| \quad (19)$$

The above objective function is subject to the following set of constraints.

– Number of PMs for the n-th transformer

$$\sum_{i\in I, k_n\in K_n} x_i^{k_n,n} = K_n, n\in N \quad (20)$$

– The k-th PM or a replacement is performed between $[i_l^{k,n}, i_u^{k,n}]$

$$\sum_{i_l^{k,n}\le i\le i_u^{k,n}} x_i^{k_n,n} = 1, k_n\in K_n, n\in N$$
$$x_i^{k,n} = 0 \text{ if } i \ne [i_l^{k,n}, i_u^{k,n}], k_n\in K_n, n\in N \quad (21)$$

– Maximum manpower availability at the i-th interval

$$\sum_{n\in N, k_n=1,\ldots,K_n-1} \gamma_n x_i^{k_n,n} + \sum_{n\in N} \beta_n x_i^{K_n,n} \le h_i, i\in I \quad (22)$$

– Maximum budget availability at the i-th interval

$$\sum_{n\in N, k_n=1,\ldots,K_n-1} \theta_n x_i^{k_n,n} + \sum_{n\in N} \rho_n x_i^{K_n,n} \le m_i, i\in I \quad (23)$$

– No replacement condition

$$x_i^{K_n,n} = 0, i\in N \text{ if } r_n > 0. \quad (24)$$

Without the global labour and budget constraints, it is easy to see that the optimal solution from the first stage is also the optimal solution for the scheduling problem, i.e., no readjustment is needed. For important transformer such that those are close to hospitals and schools, we should use the solution from the first stage, which is done by selecting a high value for w_n. Constraints in Equation (21) not only ensure the deviation is not too large so as to retain the efficient schedule from the first stage, but also help to significantly reduce the number of binary variables for the formulation. It can remove up to 90% of binary variables that makes the model attractive for a large scale system. The optimization problem in Equations (22-27) is an integer linear problem programming, which can be efficiently solved by some MILP solvers such as CPLEX [10].

5 Preliminary results case study

A planning horizon for the set of transformers of four years with one month time interval is selected in our experiment. We use CPLEX version 12.5 to solve the scheduling problem (19)-(24) and the projection subproblem arising from the gradient projection algorithm. The impact weight factor w_n is calculated based on the number of customers downstream.

First, we illustrate a typical output after each stage in Section 4.1 for four transformers. Table 1 gives the PM action results for the first stage problem where each transformer is optimized independently. Times to carry out a PM activity is reported, where only the third transformer does not need a replacement during the scheduled time horizon (the optimal planned replacements are labeled in red).

Table 1. *Optimal PMs from the first stage*

Trans	1st	2nd	3rd	4th	5th	6th
1	9	16	**22**	27	30	32
2	7	13	18	**22**	24	
3	13	**22**	30	37	42	47
4	6	10	13			

We observe that the outputs from the first stage problem violate some global constraints such as the budget availability for the month of 22nd. The second stage problem will resolve the issue by solving a system-wide problem when selecting a solution as close as possible with the solution from the first stage problem, which is achieved by the scaled objective function (19). The output given in Table 2 from the second stage problem adjusts the second PM of the third transformer for a month later, i.e., at the 23rd month, to remove the budget constraint for the period.

We also compare the total expected cost between our proposed optimized maintenance schedules with a commonly used baseline scheme, called the run-to-failure, where a periodic PM is performed. The replacement time for the periodic strategy is determined based on the excess of a threshold for the expected

Table 3. *Comparision betwen optimized schedue and baseline*

numbers of failures computed from the failure rate function. A final PM result for a set of transformers is shown in Table 3. As we can see, the baseline approach incurs a higher expected cost for all cases (in the last column). For the second transformer, there is no need a replacement during the planned time horizon, while the baseline requires a replacement, which results in a much higher cost. For the case study of 400 transformers within a 4 year plan for $14.5 million of budget, the optimized scheduled spends $13.7 million, while the baseline needs an amount of $17.6 million.

Table 2. *Optimal PMs after the second stage*

Trans	1st	2nd	3rd	4th	5th	6th
1	9	16	**22**	27	30	32
2	7	13	18	**22**	24	
3	13	**23**	30	37	42	47
4	6	10	13			

6 Conclusions and Future Work

This paper has demonstrated the potential benefits of adopting a holistic model for optimizing the maintenance and investment schedules for a large number of assets. It has been shown that for distribution transformers the holistic approach with its optimizing model may save the utility a significant amount of OPEX and CAPEX costs.

So far, a simple load model has been used, which has its limitations such as a limited knowledge of the system states and the inability to take cable losses into account. These drawbacks can be addressed by adopting a more advanced state estimation model, which can also be validated using the real-time measurements of Zaltbommel.

The transformer model has focused on aging as a function of the hot-spot temperature. However, this model does not take into account aging due to paper or oil degradation. This will be considered in future research when power transformers are modelled. For power transformers inspection measurements on oil and paper degradation are available.

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